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CP violation in the $B \to K\ell^+\ell^-$ decay

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Abstract. Standard model (SM) CP asymmetries in $B \to K\ell^+\ell^-$ are expected to be very small. This feature could help in the understanding of new physics scenarios which predict the existence of CP odd phases in various Wilson coefficients. In this paper we have analyzed the $B \to K\ell^+\ell^-$ decay in scenarios beyond the SM where the Wilson coefficients have new CP odd phases. The sensitivity of the CP asymmetries on these new weak phases is discussed.

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1 Introduction

One of the key ingredients in the standard model (SM) is CP violation, which can be described by the Cabibbo–Kobayashi–Maskawa (CKM) matrix [1–3]. However, even with this description we still have an incomplete picture concerning the origin of CP violation in the SM. The exploitation of CP violation from the theoretical and experimental sides of physics is very exciting, as it may open a window to the existence of new physics beyond the SM. Note that the existence of CP violation is a well established fact in K [4] and B [5–8] meson systems.

In order to study the sources of CP violation it is promising to consider those observables which are sensitive to the possible CP phases. For example, CP asymmetries in decay widths and lepton polarization asymmetries, such as explored in [9–25].

One of the promising directions for measuring CP violation is the analysis of rare semi-leptonic decays. From the experimental perspective the exclusive decay modes, such as $B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$, are easy to measure. Two years ago the Belle [26] and BaBar [27] collaborations announced the following results for the branching ratios for the $B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$ decays:

$$\mathrm{Br}(B \to K\ell^+\ell^-) = \begin{cases} \left(4.8^{+1.0}_{-0.9} \pm 0.3 \pm 0.1\right) \times 10^{-7} & [26]\,, \\ \left(0.65^{+0.14}_{-0.13} \pm 0.04\right) \times 10^{-6} & [27]\,, \end{cases}$$

$$\mathrm{Br}(B \to K^*\ell^+\ell^-) = \begin{cases} \left(11.5^{+2.6}_{-3.4} \pm 0.8 \pm 0.2\right) \times 10^{-7} & [26]\,, \\ \left(0.88^{+0.23}_{-0.29}\right) \times 10^{-6} & [27]\,. \end{cases}$$

The analysis for study of possible CP violation in $B \to K^*\ell^+\ell^-$ was done in earlier works [28–32]. The goal of our present work is to similarly study the possible CP violation asymmetry in the exclusive $B \to K\ell^+\ell^-$ decay using the most general form of the effective Hamiltonian, including all possible forms of interactions. There have been many attempts to study $B \to K\ell^+\ell^-$ with an extended operator basis, e.g. by Greub et al. [33]. Such an analysis will be useful for comparisons with experimental results, as the inclusive modes are generally hard to measure. Note that the CP violation in the decay $B \to K\ell^+\ell^-$ is induced by the $b \to s\ell^+\ell^-$ transition, which in the SM is practically equal to zero. This is due to the CKM factors $V_{ub}V_{us}^*$ being negligible, with the result that the unitarity condition produces only an overall phase factor in the matrix element. Therefore the CP asymmetry is strongly suppressed. As such, any deviation from zero for the CP asymmetry would be an indication of new physics.

This paper is organized as follows. In Sect. 2, using the most general form of the effective Hamiltonian, we derive the matrix element of the $B \to K\ell^+\ell^-$ decay in terms of the $B \to K$ transition form-factors. We also derive in this section the general analytic expression for the CP violating asymmetry. Section 3 contains our numerical analysis of the CP violating asymmetries together with our conclusions.

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2 The matrix element for $B \to K\ell^+\ell^-$ decay

In this section we calculate the matrix element for the $B \to K\ell^+\ell^-$ decay, which is governed by the $b \to s\ell^+\ell^-$ transition at the quark level. The matrix element for the $b \to s\ell^+\ell^-$ transition (in terms of the twelve model independent four-Fermi interactions) can be written in the following form [34, 35]:

$$\begin{aligned}\mathcal{M} = \frac{\alpha G_F}{\sqrt{2}\pi} V_{tb}V_{ts}^* \Bigg[& C_{SL}\left(\bar{s}\mathrm{i}\sigma_{\mu\nu}\frac{q^\nu}{q^2}Lb\right)\bar{\ell}\gamma^\mu\ell \\ & + C_{BR}\left(\bar{s}\mathrm{i}\sigma_{\mu\nu}\frac{q^\nu}{q^2}Rb\right)\bar{\ell}\gamma^\mu\ell + C_{LL}^{\mathrm{tot}}(\bar{s}_L\gamma_\mu b_L)\bar{\ell}_L\gamma^\mu\ell_L \\ & + C_{LR}^{\mathrm{tot}}(\bar{s}_L\gamma_\mu b_L)\bar{\ell}_R\gamma^\mu\ell_R + C_{RL}(\bar{s}_R\gamma_\mu b_R)\bar{\ell}_L\gamma^\mu\ell_L \\ & + C_{RR}(\bar{s}_R\gamma_\mu b_R)\bar{\ell}_R\gamma^\mu\ell_R + C_{LRLR}(\bar{s}_L b_R)\bar{\ell}_L\ell_R \\ & + C_{RLLR}(\bar{s}_R b_L)\bar{\ell}_L\ell_R + C_{LRRL}(\bar{s}_L b_R)\bar{\ell}_R\ell_L \\ & + C_{RLRL}(\bar{s}_R b_L)\bar{\ell}_R\ell_L + C_T\bar{s}\sigma_{\mu\nu}b\bar{\ell}\sigma^{\mu\nu}\ell \\ & + \mathrm{i}C_{TE}\varepsilon^{\mu\nu\alpha\beta}\bar{s}\sigma_{\mu\nu}b\bar{\ell}\sigma_{\alpha\beta}\ell\Bigg], \end{aligned} \tag{1}$$

where $L/R = \frac{1}{2}(1 \mp \gamma_5)$, the C_X are the Wilson coefficients of the four-Fermi interactions and $q_\mu = (p_B - p_K)_\mu = (p_+ + p_-)_\mu$ is the momentum transfer. Among the twelve Wilson coefficients several already exist in the SM. For example, the first two terms with coefficients C_{SL} and C_{BR} describe the penguin operators, where in the SM these coefficients are equal to $-2m_sC_7^{\mathrm{eff}}$ and $-2m_bC_7^{\mathrm{eff}}$. The next four terms in (1) are the vector type interactions with coefficients C_{LL}^{tot}, C_{LR}^{tot}, C_{RL} and C_{RR}. Two of these vector interactions, C_{LL}^{tot} and C_{LR}^{tot}, also exist in the SM with the form $(C_9^{\mathrm{eff}} - C_{10})$ and $(C_9^{\mathrm{eff}} + C_{10})$. Therefore we can say that the coefficients C_{LL}^{tot} and C_{LR}^{tot} describe the sum of the contributions from the SM and the new physics, where they can be written as

$$\begin{aligned} C_{LL}^{\mathrm{tot}} &= C_9^{\mathrm{eff}} - C_{10} + C_{LL}, \\ C_{LR}^{\mathrm{tot}} &= C_9^{\mathrm{eff}} + C_{10} + C_{LR}. \end{aligned} \tag{2}$$

The terms with coefficients C_{LRLR}, C_{RLLR}, C_{LRRL} and C_{RLRL} describe the scalar type interactions. The last two terms, with the coefficients C_T and C_{TE}, describe the tensor type interactions.

Now that we have (1), describing the $b \to s\ell^+\ell^-$ decay at a scale $\mu \simeq m_B$, we can write down the matrix elements for the $B \to K\ell^+\ell^-$ decay. The matrix element for this decay can be obtained by sandwiching the effective Hamiltonian between B and K meson states; these are parameterized in terms of form-factors which depend on the momentum transfer squared, $q^2 = (p_B - p_K)^2 = (p_+ - p_-)^2$. It follows from (1) that in order to calculate the amplitude of the $B \to K\ell^+\ell^-$ decay the following matrix elements are required:

$$\langle K|\bar{s}\gamma_\mu b|B\rangle, \left\langle K\left|\bar{s}\mathrm{i}\sigma_{\mu\nu}q^\nu b\right|B\right\rangle, \langle K|\bar{s}b|B\rangle, \langle K|\bar{s}\sigma_{\mu\nu}b|B\rangle.$$

These matrix elements are defined as follows [37–42]:

$$\begin{aligned} \langle K(p_K)|\bar{s}\gamma_\mu b|B(p_B)\rangle &= f_+\left[(p_B+p_K)_\mu - \frac{m_B^2 - m_K^2}{q^2}q_\mu\right] \\ &\quad + f_0\,\frac{m_B^2 - m_K^2}{q^2}\,q_\mu, \end{aligned} \tag{3}$$

$$\begin{aligned} \langle K(p_K)|\bar{s}\sigma_{\mu\nu}b|B(p_B)\rangle &= -\mathrm{i}\frac{f_T}{m_B+m_K}[(p_B+p_K)_\mu q_\nu \\ &\quad - q_\mu(p_B+p_K)_\nu]. \end{aligned} \tag{4}$$

The matrix elements $\langle K(p_K)|\bar{s}\mathrm{i}\sigma_{\mu\nu}q^\nu b|B(p_B)\rangle$ and $\langle K|\bar{s}b|B\rangle$ can be obtained from (3) and (4) by multiplying both sides of these equations by q^μ and using the equations of motion, we get

$$\langle K(p_K)|\bar{s}b|B(p_B)\rangle = f_0\frac{m_B^2 - m_K^2}{m_b - m_s}, \tag{5}$$

$$\begin{aligned} \left\langle K(p_K)\left|\bar{s}\mathrm{i}\sigma_{\mu\nu}q^\nu b\right|B(p_B)\right\rangle &= \frac{f_T}{m_B+m_K}\left[(p_B+p_K)_\mu q^2\right. \\ &\quad \left. - q_\mu(m_B^2 - m_K^2)\right]. \end{aligned} \tag{6}$$

As we have already mentioned, the form-factors entering (3)–(6) represent the hadronization process, where in order to calculate these form-factors information about the non-perturbative region of QCD is required. Therefore for the estimation of the form-factors to be reliable a non-perturbative approach is needed. Among the non-perturbative approaches the QCD sum rule [36] is more predictive in studying the properties of hadrons. The form-factors appearing in the $B \to K$ transition are computed in the framework of the three point QCD sum rules [37, 38] and in the light cone QCD sum rules [39–42]. We will use the result of the work in [42] where radiative corrections to the leading twist wave functions and $SU(3)$ breaking effects are taken into account. As a result the form-factors are parameterized in the following way [42]:

$$f_i(q^2) = \frac{r_1}{1 - q^2/m_1^2} + \frac{r_2}{\left(1 - q^2/m_1^2\right)^2}, \tag{7}$$

where $i = +$ or T, and

$$f_0(q^2) = \frac{r_2}{1 - q^2/m_{\mathrm{fit}}^2}, \tag{8}$$

with $m_1 = 5.41$ GeV and the other parameters as given in Table 1.

Table 1. The parameters for the form-factors of the $B \to K$ transition as given in [42]

	r_1	r_2	m_{fit}^2
f_+	0.162	0.173	–
f_0	0	0.33	37.46
f_T	0.161	0.198	–

Using the definition of the form-factors given in (3)–(6) we arrive at the following matrix element for the $B \to K\ell^+\ell^-$ decay:

$$
\begin{aligned}
\mathcal{M}(B \to K\ell^+\ell^-) = \frac{G_F\alpha}{4\sqrt{2}\pi} V_{tb}V_{ts}^* \{ & \bar{\ell}\gamma^\mu\ell \\
& \times \left[A(p_B+p_K)_\mu + Bq_\mu\right] \\
& + \bar{\ell}\gamma^\mu\gamma_5\ell\left[C(p_B+p_K)_\mu + Dq_\mu\right] \\
& + \bar{\ell}\ell Q + \bar{\ell}\gamma_5\ell N + 4\bar{\ell}\sigma^{\mu\nu}\ell(-\mathrm{i}G) \\
& \times \left[(p_B+p_K)_\mu q_\nu - (p_B+p_K)_\nu q_\mu\right] \\
& + 4\bar{\ell}\sigma^{\alpha\beta}\ell\,\varepsilon_{\mu\nu\alpha\beta}H\left[(p_B+p_K)_\mu q_\nu\right. \\
& \left. - (p_B+p_K)_\nu q_\mu\right]\} . \qquad (9)
\end{aligned}
$$

The functions entering (9) are defined by

$$
\begin{aligned}
A &= (C_{LL}^{\mathrm{tot}} + C_{LR}^{\mathrm{tot}} + C_{RL} + C_{RR})f_+ \\
&\quad + 2(C_{BR} + C_{SL})\frac{f_T}{m_B+m_K} , \\
B &= (C_{LL}^{\mathrm{tot}} + C_{LR}^{\mathrm{tot}} + C_{RL} + C_{RR})f_- \\
&\quad - 2(C_{BR} + C_{SL})\frac{f_T}{(m_B+m_K)q^2}\left(m_B^2 - m_K^2\right) , \\
C &= (C_{LR}^{\mathrm{tot}} + C_{RR} - C_{LL}^{\mathrm{tot}} - C_{RL})f_+ , \\
D &= (C_{LR}^{\mathrm{tot}} + C_{RR} - C_{LL}^{\mathrm{tot}} - C_{RL})f_- , \\
Q &= f_0\frac{m_B^2 - m_K^2}{m_b - m_s}(C_{LRLR} + C_{RLLR} + C_{LRRL} + C_{RLRL}) , \\
N &= f_0\frac{m_B^2 - m_K^2}{m_b - m_s}(C_{LRLR} + C_{RLLR} - C_{LRRL} - C_{RLRL}) , \\
G &= \frac{C_T}{m_B+m_K}f_T , \\
H &= \frac{C_{TE}}{m_B+m_K}f_T , \qquad (10)
\end{aligned}
$$

where

$$
f_- = (f_0 - f_+)\frac{m_B^2 - m_K^2}{q^2} .
$$

From (9) it follows that the difference from the SM is due to the last four terms only, namely the scalar and tensor type interactions. For an analysis of the CP asymmetry it is necessary to compute the differential decay width for $B \to K\ell^+\ell^-$. From the expression of the matrix element given in (9) we calculate the following result for the dilepton invariant mass spectrum for $B \to K\ell^+\ell^-$:

$$
\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}} = \frac{G_F^2\alpha^2 m_B}{2^{14}\pi^5}|V_{tb}V_{ts}^*|^2\lambda^{1/2}(1,\hat{r}_K,\hat{s})v\Delta(\hat{s}) , \qquad (11)
$$

where $\lambda(1,\hat{r}_K,\hat{s}) = 1 + \hat{r}_K^2 + \hat{s}^2 - 2\hat{r}_K - 2\hat{s} - 2\hat{r}_K\hat{s}$, $\hat{s} = q^2/m_B^2$, $\hat{r}_K = m_K^2/m_B^2$, $\hat{m}_\ell = m_\ell/m_B$, $v = \sqrt{1 - 4\hat{m}_\ell^2/\hat{s}}$ is the final lepton velocity, and $\Delta(\hat{s})$ is

$$
\begin{aligned}
\Delta = \frac{4m_B^2}{3}\,\mathrm{Re}\,\big[& -96\lambda m_B^3\hat{m}_\ell(AG^*) \\
& + 24m_B^2\hat{m}_\ell^2(1-\hat{r}_K)(CD^*) + 12m_B\hat{m}_\ell(1-\hat{r}_K)(CN^*) \\
& + 12m_B^2\hat{m}_\ell^2\hat{s}|D|^2 + 3\hat{s}|N|^2 + 12m_B\hat{m}_\ell\hat{s}(DN^*) \\
& + 256\lambda m_B^4\hat{s}v^2|H|^2 + \lambda m_B^2(3-v^2)|A|^2 + s3\hat{s}v^2|Q|^2 \\
& + 64\lambda m_B^4\hat{s}(3-2v^2)|G|^2 \\
& + m_B^2\left\{2\lambda - (1-v^2)\left[2\lambda - 3(1-\hat{r}_K)^2\right]\right\}|C|^2\big] . \qquad (12)
\end{aligned}
$$

As we have already mentioned, our goal in this work is the study of possible CP violating asymmetries beyond the SM in the $B \to K\ell^+\ell^-$ decay; at this point we shall briefly remind the reader of the situation in the SM. In the SM the C_9 Wilson coefficient is the only one to have strong and weak phases. Strong phases arise from the short distance effects and resonances whereas the weak phase comes from the CKM elements. It is well known that the Wilson coefficient C_7 gets a strong phase when next and next to next leading order QCD corrections are taken into account in the SM [43]. But even after taking these corrections into account the Wilson coefficient C_{10} still remains real in the SM. From the parameterization of the form-factors it follows that they are inherently real, and thus the imaginary parts in the functions in (12) can come only from the Wilson coefficients in (1). By *strong* and *weak* phases we mean the phases which are CP even and odd respectively. In other words we shall consider the picture where CP violating effects due to the short distance dynamics are parameterized by the Wilson coefficients. In principle all Wilson coefficients can have non-zero strong and weak phases. In general the amplitude for $\bar{B} \to K$ has the general form [29–32]

$$
A(\bar{B} \to K) = e^{\mathrm{i}\varphi_1}A_1e^{\mathrm{i}\delta_1} + e^{\mathrm{i}\varphi_2}A_2e^{\mathrm{i}\delta_2} , \qquad (13)
$$

where the strong phases are labeled as δ and the weak phases by φ. As noted above the strong phases are CP even, whereas weak phases are odd under CP. Thus we arrive at an amplitude for the conjugated process, $B \to \bar{K}$, from (13):

$$
\bar{A}(B \to \bar{K}) = e^{-\mathrm{i}\varphi_1}A_1e^{\mathrm{i}\delta_1} + e^{-\mathrm{i}\varphi_2}A_2e^{\mathrm{i}\delta_2} , \qquad (14)
$$

where the amplitudes of the decay rate of particle and antiparticle can be defined by the CP asymmetry (in the decay rate) as

$$
\begin{aligned}
A_{CP} &= \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} \\
&= \frac{-2A_1A_2\sin(\varphi_1-\varphi_2)\sin(\delta_1-\delta_2)}{A_1^2 + 2A_1A_2\cos(\varphi_1-\varphi_2)\cos(\delta_1-\delta_2) + A_2^2} . \qquad (15)
\end{aligned}
$$

Note that from the above expression we observe that in order to have CP asymmetry we should have both strong and weak phases in the amplitude, where the strong phases are provided by C_9^{eff}. In the SM the weak phases for the $b \to s\ell^+\ell^-$ transition are negligible and hence the CP

asymmetry for processes based on the quark level transitions, $b\to s\ell^+\ell^-$, are highly suppressed. We will now consider the CP asymmetry in the decay width which is defined as

$$A_{CP}(q^2)=\frac{\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}}(\bar{B}\to K\ell^+\ell^-)-\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}}(B\to \bar{K}\ell^+\ell^-)}{\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}}(\bar{B}\to K\ell^+\ell^-)+\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}}(B\to \bar{K}\ell^+\ell^-)}\,. \quad (16)$$

Note that one can also have a CP asymmetry from the forward–backward (FB) asymmetry [61, 62]. However, in our present case, the FB asymmetry for $B\to K\ell^+\ell^-$ vanishes within the SM.

We shall now consider the minimal extension of these Wilson coefficients. In this approach we shall assume that the Wilson coefficients corresponding to scalar and tensor type interactions vanish identically (of course in the general case we can consider all Wilson coefficients with an arbitrary weak phase). For scalar type operators which emerge in supersymmetric (SUSY) models and two Higgs doublet models (2HDM) this assumption is justified when we have electrons or muons in the final state. The reason being that in SUSY and 2HDM these operators originate from an Higgs exchange which results in Wilson coefficients which are proportional to m_ℓ, and which hence are negligible for $\ell=e,\mu$.

The Wilson coefficients for the dipole operator obey

$$C_{BR}=-2C_7^{\mathrm{eff}}m_b\,,\quad C_{SL}=-2C_7^{\mathrm{eff}}m_s\,, \quad (17)$$

with

$$C_7^{\mathrm{eff}}=|C_7^{\mathrm{eff}}|\exp(\mathrm{i}\varphi_7)\,,$$

where φ_7 is an arbitrary phase not constrained by the already observed branching ratio $\mathrm{Br}(B\to K^*\gamma)$.

Regarding the appearance of the new weak phase in C_{10} we feel that a few words are in order. One of the possible discrepancies between the experimental results [44–46] and the theoretical prediction for $B\to\pi K$ (from the $B\to\pi\pi$ data) can be resolved, as proposed in [47–53], by introducing a complex phase in the Wilson coefficient $C_{10}=C_{10}^{\mathrm{SM}}\exp(\mathrm{i}\varphi_{10})$. In this prescription the weak phase given to C_{10} does not affect the CP asymmetry in $B\to K\ell^+\ell^-$.

We will assume that the Wilson coefficients C_{RL} and C_{RR} also have weak phases; that is,

$$\begin{aligned}C_{RL}&=|C_{RL}|\exp(\mathrm{i}\varphi_{RL})\,,\\ C_{RR}&=|C_{RR}|\exp(\mathrm{i}\varphi_{RR})\,.\end{aligned} \quad (18)$$

The Wilson coefficient $C_9^{\mathrm{eff}}(m_b,q^2)$ has a finite phase, where, in order to better appreciate this, we write its explicit phase content as

$$\begin{aligned}C_9^{\mathrm{eff}}(m_b)=&\,C_9(m_b)\left\{1+\frac{\alpha_\mathrm{s}(\mu)}{\pi}\omega(\hat{s})\right\}+Y_{\mathrm{SD}}\,(m_b,\hat{s})\\ &+Y_{\mathrm{LD}}(m_b,\hat{s})\,,\end{aligned} \quad (19)$$

where $C_9(m_b)=4.334$. Here $\omega\,(\hat{s})$ represents the $\mathcal{O}(\alpha_\mathrm{s})$ corrections coming from the four quark operator $\mathcal{O}_9$ [54, 55]:

$$\begin{aligned}\omega(\hat{s})=&-\frac{2}{9}\pi^2-\frac{4}{3}\mathrm{Li}_2(\hat{s})-\frac{2}{3}\ln(\hat{s})\ln(1-\hat{s})\\ &-\frac{5+4\hat{s}}{3(1+2\hat{s})}\ln(1-\hat{s})-\frac{2\hat{s}(1+\hat{s})(1-2\hat{s})}{3(1-\hat{s})^2(1+2\hat{s})}\ln(\hat{s})\\ &+\frac{5+9\hat{s}-6\hat{s}^2}{3(1-\hat{s})(1+2\hat{s})}\,.\end{aligned} \quad (20)$$

In (19) Y_{SD} and Y_{LD} represent, respectively, the short- and long-distance contributions to the four quark operators $\mathcal{O}_{i=1,\cdots,6}$ [54–56]. Here Y_{SD} can be obtained by a perturbative calculation:

$$\begin{aligned}Y_{\mathrm{SD}}(m_b,\hat{s})=&\,g(\hat{m}_c,\hat{s})[3C_1+C_2+3C_3+C_4+3C_5+C_6]\\ &-\frac{1}{2}g(1,\hat{s})[4C_3+4C_4+3C_5+C_6]\\ &-\frac{1}{2}g(0,\hat{s})[C_3+3C_4]+\frac{2}{9}[3C_3+C_4+3C_5+C_6]\\ &-\frac{V_{us}^*V_{ub}}{V_{ts}^*V_{tb}}[3C_1+C_2][g(0,\hat{s})-g(\hat{m}_c,\hat{s})]\,,\end{aligned} \quad (21)$$

where the loop function $g(m_q,s)$ represents the loops of quarks with mass m_q at the dilepton invariant mass s. This function develops absorptive parts for dilepton energies $s=4m_q^2$:

$$\begin{aligned}g(\hat{m}_q,\hat{s})=&-\frac{8}{9}\ln\hat{m}_q+\frac{8}{27}+\frac{4}{9}y_q-\frac{2}{9}(2+y_q)\sqrt{|1-y_q|}\\ &\times\left\{\Theta(1-y_q)\left(\ln\frac{1+\sqrt{1-y_q}}{1-\sqrt{1-y_q}}-\mathrm{i}\pi\right)\right.\\ &\left.+\Theta(y_q-1)2\arctan\frac{1}{\sqrt{y_q-1}}\right\},\end{aligned} \quad (22)$$

where $\hat{m}_q=m_q/m_b$ and $y_q=4\hat{m}_q^2/\hat{s}$. Therefore, due to the extension of the absorptive parts of $g(\hat{m}_q,\hat{s})$ we see that the strong phases come from Y_{SD}. In particular one notices that the terms proportional to $g(0,\hat{s})$ have a non-vanishing imaginary part, independent of the dilepton invariant mass.

In addition to these perturbative contributions the $\bar{c}c$ loops can excite low-lying charmonium states $\psi(1s),\ldots,\psi(6s)$ whose contributions are represented by Y_{LD}[1] [57–59]:

$$\begin{aligned}Y_{\mathrm{LD}}(m_b,\hat{s})=&\,\frac{3}{\alpha^2}\left\{-\frac{V_{cs}^*V_{cb}}{V_{ts}^*V_{tb}}C^{(0)}-\frac{V_{us}^*V_{ub}}{V_{ts}^*V_{tb}}\right.\\ &\left.\times[3C_3+C_4+3C_5+C_6]\right\}\\ &\times\sum_{V_i=\psi(1s),\ldots,\psi(6s)}\frac{\pi\kappa_i\Gamma(V_i\to\ell^+\ell^-)M_{V_i}}{(M_{V_i}^2-\hat{s}m_b^2-\mathrm{i}M_{V_i}\Gamma_{V_i})}\,,\end{aligned} \quad (23)$$

[1] The long-distance effects can also be taken into account using the prescription given in [43], but we in our analysis have used the prescription given by Kruger and Sehgal [17, 18, 57–59].

where κ_i is a phenomenological parameter taken here to be 2.3 so as to produce the correct branching ratio of $\mathrm{Br}(B \to J/\psi K^* \to K^*\ell\ell) = \mathrm{Br}(B \to J/\psi K^*)\mathrm{Br}(J/\psi \to \ell\ell)$ [17, 18], and $C^{(0)} \equiv 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6 = 0.362$. Contrary to Y_{SD} the long-distance contribution in Y_{LD} has both weak and strong phases. The weak phases follow from the CKM elements whereas the strong phases come from the $\hat{s}$ values for which the ith charmonium states are on shell. Therefore, the Wilson coefficient $C_9^{\mathrm{eff}}(m_b)$ has both weak and strong phases already in the SM.

3 Numerical analysis

In this section we present our numerical results for the asymmetries A_{CP} for the $B \to K\mu^+\mu^-$ decay. Note that the parameters for the hadronic form-factors are taken from Table 1. For values of the Wilson coefficients in the SM we have used $C_3 = 0.011$, $C_4 = -0.026$, $C_5 = 0.007$, $C_6 = -0.031$, $C_7^{\mathrm{eff}} = -0.313$, $C_9 = 4.344$, and $C_{10} = -4.664$. For further numerical analysis the values of the new Wilson coefficients are needed, where we have varied them in the range $-|C_{10}| < C_X < |C_{10}|$. The experimental value of the branching ratio of the $B \to K(K^*)\ell^+\ell^-$ decays [26, 27] and the bound on $\mathrm{Br}(B \to \mu^+\mu^-)$ [60] suggest that this is the right order of magnitude. It should be noted that the experimental results lead to strong restrictions on some of the Wilson coefficients, namely $-2 \leq C_{LL}$ and $C_{RL} \leq 2.3$, while the remaining coefficients vary in the range $-|C_{10}| < C_X < |C_{10}|$. For the remaining parameters we take $m_b = 4.8$ GeV, $m_c = 1.35$ GeV, $m_B = 5.28$ GeV and $m_K = 0.496$ GeV.

For the kinematical interval the dilepton invariant mass is $4m_\ell^2 \leq q^2 \leq (m_B - m_K)^2$, where the J/ψ family of resonances can be excited. The dominant contribution comes from the three low-lying resonances $J/\psi, \psi', \psi''$ in the interval $8\,\mathrm{GeV}^2 \lesssim q^2 \lesssim 14.5\,\mathrm{GeV}^2$. In order to minimize the hadronic uncertainties we will discard this subinterval in the analysis below by dividing the q^2 region into *low* and *high* dilepton mass intervals:

$$\begin{aligned} &\text{Region I}: \quad 4m_\ell^2 \leq q^2 \leq 8\,\mathrm{GeV}^2\,, \\ &\text{Region II}: \quad 14.5\,\mathrm{GeV}^2 \leq q^2 \leq (m_B - m_K)^2\,, \end{aligned} \tag{24}$$

where the contribution of the higher resonances do still exist in the second region.

As mentioned previously, we have analyzed the case where there are four weak phases: φ_7, φ_{10}, φ_{RL} and φ_{RR}.

In Fig. 1 we have presented A_{CP} in the φ_7–q^2 plane for the $B \to K\mu^+\mu^-$ decay for Region I and Region II, respectively. In Region I the CP asymmetry is practically independent of q^2, becoming maximal in the value for CP violation at $\varphi_7 = \pi/2$. In Region II, however, the q^2 dependence is comparatively enhanced as the dominance of the dipole coefficient is now reduced. Aside from this our figures suggest that the CP asymmetry in Region II is four times larger than in Region I, and this confirms our earlier expectation.

Since the CP asymmetry is dependent on q^2 and the new weak phases there can appear some difficulties. The

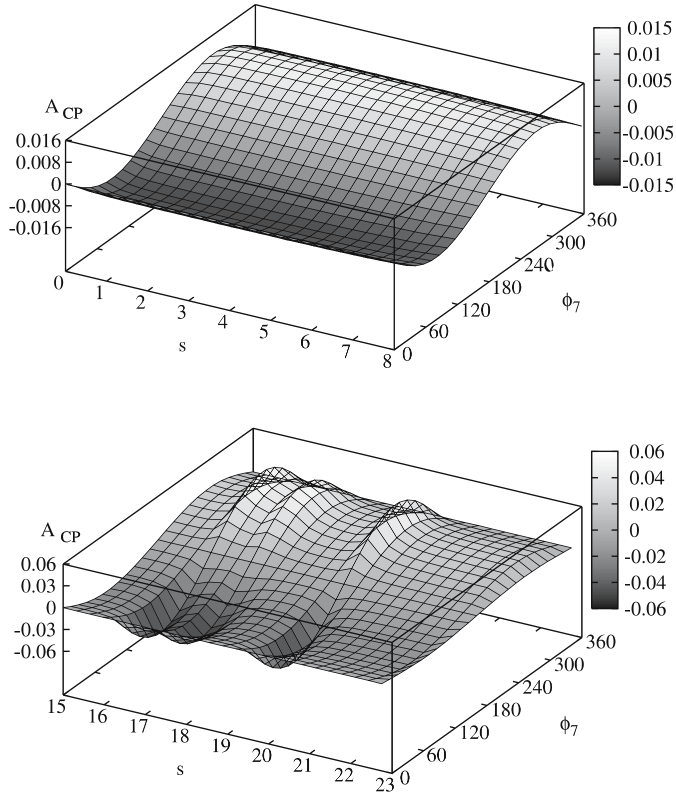

Fig. 1. Plot of the CP asymmetry in $B \to K\mu^+\mu^-$ as a function of the phase of $C_7(\varphi_7)$ and the dilepton invariant mass. The *upper plot* is for Region I and the *plot below* is for Region II. The other Wilson coefficients are taken to have their SM values

dependence of one of the variables, for example q^2, can be removed by integrating over q^2 in the allowed practical kinematical region, where the averaged asymmetries could be measured more easily experimentally. Therefore we shall now discuss only averaged CP asymmetries, which we define in the following way. That is, our averaging procedure is defined by

$$\langle A_{CP} \rangle = \frac{\displaystyle\int_{R_i} A_{CP} \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} \mathrm{d}q^2}{\displaystyle\int_{R_i} \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} \mathrm{d}q^2}\,. \tag{25}$$

where R_i means Region I or II.

We now depict in Fig. 2 the φ_7 dependence of the averaged asymmetries $\langle A_{CP} \rangle$. From this figure it can be observed that the average $\langle A_{CP} \rangle$ asymmetry can attain values of 3%. Differences from zero of any value of $\langle A_{CP} \rangle$ would be an unambiguous indication of the existence physics beyond the SM.

In Fig. 3 we have plotted the dependence of CP asymmetry on the dilepton invariant mass and φ_{RL}. In Fig. 5 we have shown the same kind of plot but for φ_{RR}. We have also shown the correlation of averaged CP asymmetry and the integrated branching ratios. In Fig. 4 the variation of $\langle A_{CP} \rangle$ with integrated branching ratio for $B \to K\mu^+\mu^-$ for C_{RL} is shown. In this figure we have used three different

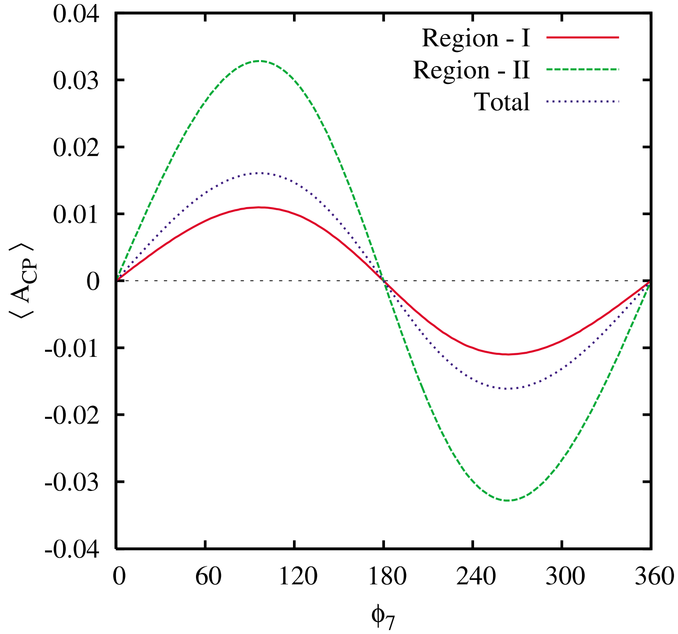

Fig. 2. The averaged CP asymmetry ($\langle A_{CP} \rangle$) in the $B \to K\mu^+\mu^-$ decay, where C_7 has a phase. The remaining Wilson coefficients are taken to have their SM values

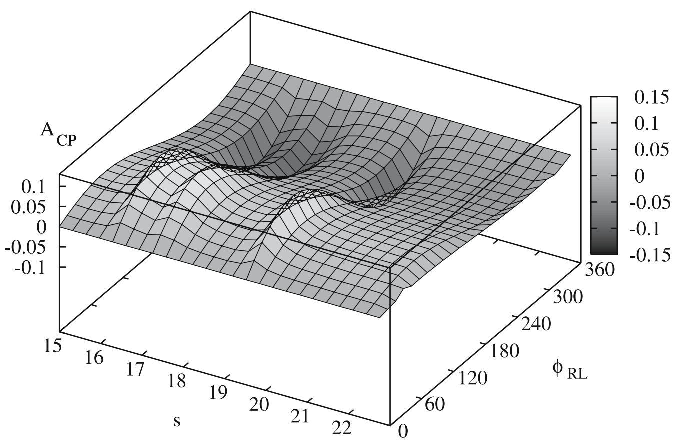

Fig. 3. A plot of the CP asymmetry in $B \to K\mu^+\mu^-$ as a function of the phase φ_{RL} and the dilepton invariant mass. In this plot we have taken $|C_{RL}| = 2$, the other Wilson coefficients having their SM values

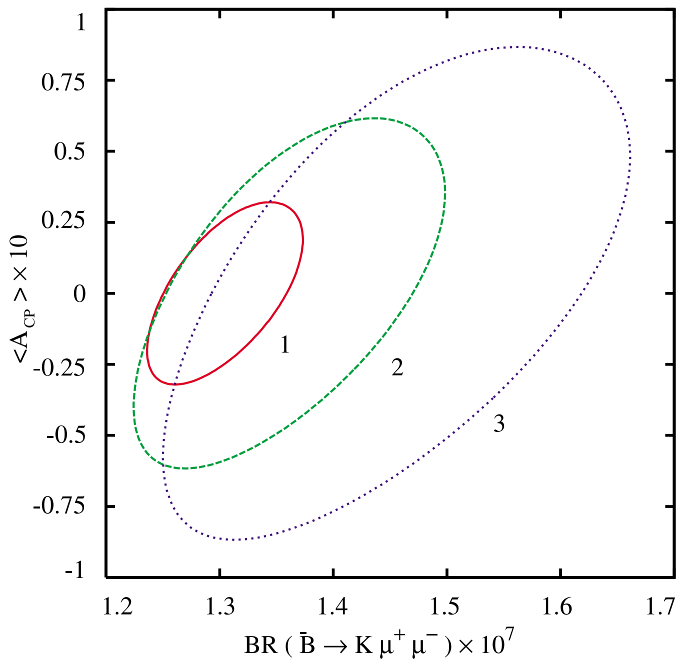

Fig. 4. The averaged CP asymmetry ($\langle A_{CP} \rangle$) in the $B \to K\mu^+\mu^-$ decay against the partial branching fraction of $B \to K\mu^+\mu^-$. In this plot we have taken various values of magnitudes of C_{RL} (as stated in the figure) and varied the phase in the range $0 \leq \varphi_{RL} \leq 2\pi$. We have plotted only Region II here

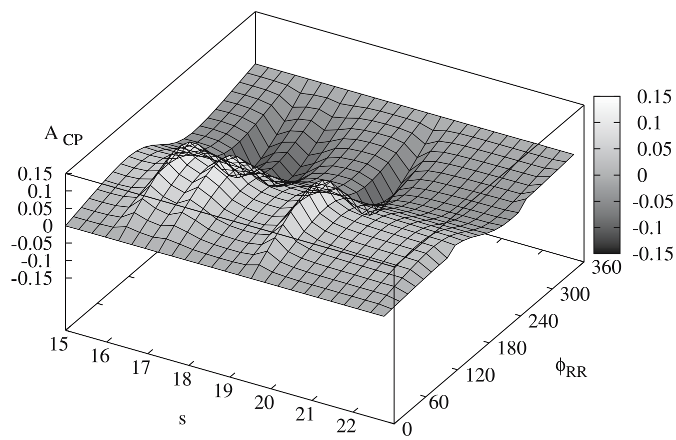

Fig. 5. A plot of the CP asymmetry in $B \to K\mu^+\mu^-$ as a function of the phase φ_{RR} and the dilepton invariant mass. In this plot we have taken $|C_{RR}| = 2$, the other Wilson coefficients having their SM values

values of C_{RL} and have varied the phase (φ_{RL}) in the range $0 \leq \varphi_{RL} \leq 2\pi$. All the other Wilson coefficients are taken to have their SM values. In a similar graph, given in Fig. 6, we have varied C_{RR}.

In the present work we have studied the sensitivity of the CP violation asymmetry on the new weak phases appearing in the Wilson coefficients. We have also observed that the CP asymmetry in Region II is 4–5 times larger than that observed in Region I when we consider a weak phase φ_7. This can be understood in that in Region II contributions coming from other operators become comparable with the dipole operator O_7, where this operator is dominant in Region I. Having obtained the averaged $\langle A_{CP} \rangle$ asymmetry we obtained a maximal value of approximately 3%. Note that an additional weak phase in C_{10} will not give rise to any CP asymmetry; however, if non-standard[2] electroweak operators are considered, then the CP asymmetry in the region of high dilepton invariant mass can reach a value of up to 10%.

[2] By non-standard we mean operators which are not present within the SM.

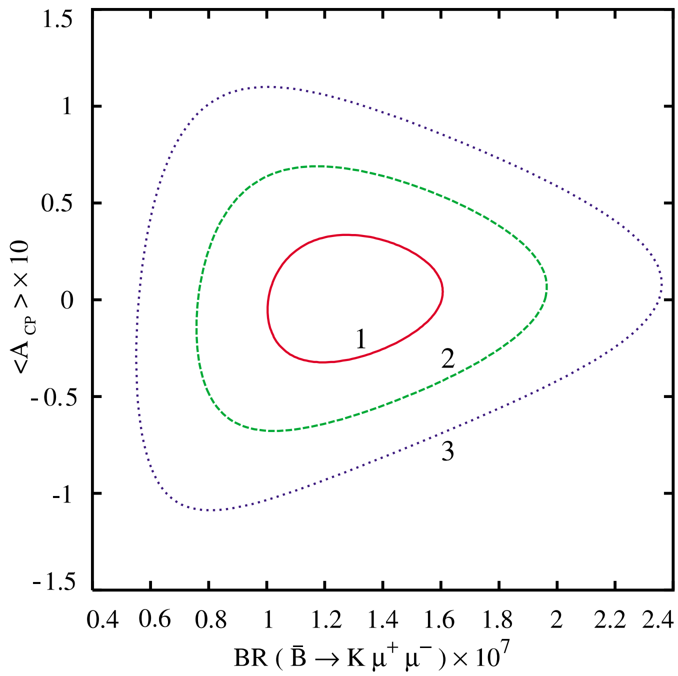

Fig. 6. The averaged CP asymmetry ($\langle A_{CP} \rangle$) in the $B \to K\mu^+\mu^-$ decay against the partial branching fraction of $B \to K\mu^+\mu^-$. In this plot we have taken various values of magnitudes of C_{RR} (as stated in the figure) and varied the phase in the range $0 \le \varphi_{RR} \le 2\pi$. We have plotted only Region II here

The analysis we have presented above is based on the most general effective Hamiltonian. All the models such as SM, two Higgs doublet model (2HDM), supersymmetry (SUSY) etc. are special cases of our analysis. In the SM all the new operators[3] $C_{LL} = C_{RL} = C_{LR} = C_{RR} = 0$. Various extensions of the SM give non-zero values to these operators. In minimal flavor violating (MFV) models[4] we can in general have non-zero values of C_{LL} and C_{RL}. However, in general, models like SUSY and LR models can give various new chirality operators like C_{LR} and C_{RR}. The CP asymmetries we have presented above, if observed, will give clear signatures of the presence of CP violation in an extended operators basis.

Let us try to estimate how many $B\bar{B}$ pairs would be required to measure the asymmetries we have presented above. Experimentally to observe an asymmetry $\mathcal{A}$ at $n\sigma$ of a decay, with branching ratio $\mathcal{B}$, the number of events ($B\bar{B}$ pairs here) required is

$$N = \frac{n^2}{\mathcal{B}\mathcal{A}^2} \, .$$

Assuming the detection efficiency of μ is 100%, we will require this to be $\sim 10^{10}$ events for a 3σ measurement. This kind of measurement might not be feasible in present generation B-factories but can be done at future LHCb and super-B experiments.

As stated earlier, we can also, in principle, have weak phases in scalar and pseudo-scalar operators. The presence of weak phases in these operators can also substantially affect the CP asymmetry. The popular extensions of the SM, such as SUSY and 2HDM, all predict the existence of such operators. However, the magnitude of these Wilson coefficients is predicted to be small when the lepton $\ell = e$ or μ. In the presence of these operators one also gets a non-zero value for the FB asymmetry in $B \to K\ell^+\ell^-$. The FB asymmetry could provide another measure of CP asymmetry [61, 62] which has not been considered in this work.

The observation of CP asymmetry in $B \to K\ell^+\ell^-$ would not only tell us about the nature of weak phases but would also give us insight in the structure of the effective Hamiltonian. Therefore the measurement of the CP violating asymmetry would provide us with useful insight into the mechanism of CP violation, which in turn would serve as a good test for physics beyond the SM.

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References

1. N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963)
2. J.L. Hewett, J.D. Wells, Phys. Rev. D **55**, 5549 (1997) [hep-ph/9610323]
3. M. Kobayashi, T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973)
4. J.H. Christenson, J.W. Cronin, V.L. Fitch, R. Turlay, Phys. Rev. Lett. **13**, 138 (1964)
5. BaBar Collaboration, B. Aubert et al., Phys. Rev. Lett. **87**, 091 801 (2001) [hep-ex/0107013]
6. BaBar Collaboration, B. Aubert et al., Phys. Rev. Lett. **93**, 131 801 (2004) [hep-ex/0407057]
7. Belle Collaboration, K. Abe et al., Phys. Rev. Lett. **87**, 091 802 (2001) [hep-ex/0107061]
8. Belle Collaboration, Y. Chao et al., Phys. Rev. Lett. **93**, 191 802 (2004) [hep-ex/0408100]
9. C.H. Chen, C.Q. Geng, Phys. Rev. D **66**, 094 018 (2002) [hep-ph/0209352]
10. T.M. Aliev, H. Koru, A. Ozpineci, M. Savci, Phys. Lett. B **400**, 194 (1997) [hep-ph/9702209]
11. T.M. Aliev, M. Savci, A. Ozpineci, H. Koru, J. Phys. G **24**, 49 (1998) [hep-ph/9705222]
12. T.M. Aliev, M.K. Cakmak, M. Savci, Nucl. Phys. B **607**, 305 (2001) [hep-ph/0009133]
13. T.M. Aliev, M. Savci, Phys. Lett. B **481**, 275 (2000) [hep-ph/0003188]
14. T.M. Aliev, V. Bashiry, M. Savci, JHEP **0405**, 037 (2004) [hep-ph/0403282]
15. T.M. Aliev, A. Ozpineci, M. Savci, C. Yuce, Phys. Rev. D **66**, 115 006 (2002) [hep-ph/0208128]
16. T.M. Aliev, M.K. Cakmak, A. Ozpineci, M. Savci, Phys. Rev. D **64**, 055 007 (2001) [hep-ph/0103039]

[3] The definition of the operators is as in (1) and (2).

[4] We define MFV as in [64].

17. F. Kruger, L.M. Sehgal, Phys. Lett. B **380**, 199 (1996) [hep-ph/9603237]
18. J.L. Hewett, Phys. Rev. D **53**, 4964 (1996) [hep-ph/9506289]
19. T.M. Aliev, A. Ozpineci, M. Savci, Phys. Lett. B **511**, 49 (2001) [hep-ph/0103261]
20. T.M. Aliev, A. Ozpineci, M. Savci, Nucl. Phys. B **585**, 275 (2000) [hep-ph/0002061]
21. T.M. Aliev, C.S. Kim, Y.G. Kim, Phys. Rev. D **62**, 014 026 (2000) [hep-ph/9910501]
22. N. Gaur, hep-ph/0305242
23. S.R. Choudhury, N. Gaur, A.S. Cornell, G.C. Joshi, Phys. Rev. D **68**, 054 016 (2003) [hep-ph/0304084]
24. S. Rai Choudhury, A. Gupta, N. Gaur, Phys. Rev. D **60**, 115 004 (1999) [hep-ph/9902355]
25. A.S. Cornell, N. Gaur, JHEP **0309**, 030 (2003) [hep-ph/0308132]
26. Belle Collaboration, A. Ishikawa et al., Phys. Rev. Lett. **91**, 261 601 (2003) [hep-ex/0308044]
27. BaBar Collaboration, B. Aubert et al., Phys. Rev. Lett. **91**, 221 802 (2003) [hep-ex/0308042]
28. T.M. Aliev, D.A. Demir, M. Savci, Phys. Rev. D **62**, 074 016 (2000) [hep-ph/9912525]
29. F. Kruger, E. Lunghi, Phys. Rev. D **63**, 014 013 (2001) [hep-ph/0008210]
30. F. Kruger, J.C. Romao, Phys. Rev. D **62**, 034 020 (2000) [hep-ph/0002089]
31. A.S. Cornell, N. Gaur, JHEP **0502**, 005 (2005) [hep-ph/0408164]
32. A.S. Cornell, N. Gaur, S.K. Singh, hep-ph/0505136
33. C. Greub, A. Ioannisian, D. Wyler, Phys. Lett. B **346**, 149 (1995) [hep-ph/9408382]
34. S. Fukae, C.S. Kim, T. Yoshikawa, Phys. Rev. D **61**, 074 015 (2000) [hep-ph/9908229]
35. S. Fukae, C.S. Kim, T. Morozumi, T. Yoshikawa, Phys. Rev. D **59**, 074 013 (1999) [hep-ph/9807254]
36. M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B **147**, 385 (1979)
37. P. Colangelo, F. De Fazio, P. Santorelli, E. Scrimieri, Phys. Rev. D **53**, 3672 (1996)
38. P. Colangelo, F. De Fazio, P. Santorelli, E. Scrimieri, Phys. Rev. D **57**, 3186 (1998) [Erratum] [hep-ph/9510403]
39. P. Ball, R. Zwicky, JHEP **0110**, 019 (2001) [hep-ph/0110115]
40. P. Ball, JHEP **9809**, 005 (1998) [hep-ph/9802394]
41. T.M. Aliev, H. Koru, A. Ozpineci, M. Savci, Phys. Lett. B **400**, 194 (1997) [hep-ph/9702209]
42. P. Ball, R. Zwicky, Phys. Rev. D **71**, 014 015 (2005) [hep-ph/0406232]
43. B. Grinstein, D. Pirjol, Phys. Rev. D **70**, 114 005 (2004) [hep-ph/0404250]
44. Belle Collaboration, Y. Chao et al., Phys. Rev. D **69**, 111 102 (2004) [hep-ex/0311061]
45. BaBar Collaboration, B. Aubert et al., Phys. Rev. Lett. **89**, 281 802 (2002) [hep-ex/0207055]
46. BaBar Collaboration, B. Aubert et al., hep-ex/0408080
47. A.J. Buras, R. Fleischer, Eur. Phys. J. C **16**, 97 (2000) [hep-ph/0003323]
48. A.J. Buras, R. Fleischer, S. Recksiegel, F. Schwab, Eur. Phys. J. C **32**, 45 (2003) [hep-ph/0309012]
49. A.J. Buras, R. Fleischer, S. Recksiegel, F. Schwab, Nucl. Phys. B **697**, 133 (2004) [hep-ph/0402112]
50. A.J. Buras, R. Fleischer, S. Recksiegel, F. Schwab, Phys. Rev. Lett. **92**, 101 804 (2004) [hep-ph/0312259]
51. T. Yoshikawa, Phys. Rev. D **68**, 054 023 (2003) [hep-ph/0306147]
52. M. Gronau, J.L. Rosner, Phys. Lett. B **572**, 43 (2003) [hep-ph/0307095]
53. M. Beneke, M. Neubert, Nucl. Phys. B **675**, 333 (2003) [hep-ph/0308039]
54. M. Misiak, Nucl. Phys. B **393**, 23 (1993)
55. M. Misiak, Nucl. Phys. B **439**, 461 (1995) [Erratum]
56. A.J. Buras, M. Munz, Phys. Rev. D **52**, 186 (1995) [hep-ph/9501281]
57. N.G. Deshpande, J. Trampetic, K. Panose, Phys. Rev. D **39**, 1461 (1989)
58. C.S. Lim, T. Morozumi, A.I. Sanda, Phys. Lett. B **218**, 343 (1989)
59. A. Ali, G. Hiller, L.T. Handoko, T. Morozumi, Phys. Rev. D **55**, 4105 (1997) [hep-ph/9609449]
60. D0 Collaboration, V.M. Abazov et al., Phys. Rev. Lett. **94**, 071 802 (2005) [hep-ex/0410039]
61. S.R. Choudhury, Phys. Rev. D **56**, 6028 (1997) [hep-ph/9706313]
62. G. Buchalla, G. Hiller, G. Isidori, Phys. Rev. D **63**, 014 015 (2001) [hep-ph/0006136]
63. Y. Rodriguez, C. Quimbay, hep-ph/0301251
64. A.J. Buras, Acta Phys. Pol. B **34**, 5615 (2003) [hep-ph/0310208]